The Thermal Evolution of Ultramagnetized Neutron Stars

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ABSTRACT

Using recently calculated analytic and numerical models for the thermal structure of ultramagnetized neutron stars, we estimate the effects that ultrastrong magnetic fields $B \geq 10^{14}$ G have on the thermal evolution of a neutron star. Understanding this evolution is necessary to interpret models that invoke "magnetars" to account for soft γ-ray emission from some repeating sources.

Subject headings: stars: neutron — stars: magnetic fields — radiative transfer — X-rays: stars

1. Introduction

Neutron stars with extremely strong magnetic dipole fields ($B \gtrsim 10^{14}$ G) may form if a helical dynamo mechanism operates efficiently during the first few seconds after gravitational collapse (Thompson & Duncan 1993) or through the conventional process of flux freezing if the progenitor star has a sufficiently intense core field. These "magnetars" initially rotate with periods $P \sim 1$ ms, but would quickly slow down due to magnetic dipole radiation and cross the pulsar death line after about $10^5 - 10^6$ yr. With their strong magnetic fields, magnetars have been used to explain several phenomena including gamma-ray bursts (Usov 1992, Duncan & Thompson 1992) and soft gamma repeaters (Thompson & Duncan 1995).

Shibanov & Yakovlev (1996) have recently discussed how magnetic fields $B < 10^{13.5}$ G affect neutron star cooling. (Throughout, we will use the symbol B to denote the field strength at the magnetic pole.) They find that below $B \sim 10^{12}$ G, the magnetic field suppresses the total heat flux radiated by a neutron star. However, for $B \gtrsim 10^{12}$ G, the quantization of the electron energies enhances the conductivity along the field lines, resulting in a net increase in the heat flux. Here, we extend these results into the ultramagnetized regime with $B = 10^{14} - 10^{16}$ G.

In this *Letter*, we will discuss the cooling evolution of neutron stars which have not accreted significant material from their surroundings, *i.e.* neutron stars with iron envelopes.

2. Model Envelopes

Heyl & Hernquist (1997) have developed analytic models for ultramagnetized neutron star envelopes and find that the transmitted flux through the envelope is simply related to the direction and strength of the magnetic field and to the core temperature (T_c). Using these results as a guide, we numerically

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integrate several envelopes with $B = 10^{14} - 10^{16}$ G for the case of parallel transport. We will present the detailed results of these calculations in a future article.

At the outer boundary, we apply the photospheric condition (*e.g.* Kippenhahn & Weigert 1990). In the non-degenerate regime, photon conduction dominates. For the range of effective temperatures considered free-free absorption is the most important source of opacity, and we estimate the anisotropy factor due to the magnetic field using the results of Pavlov & Panov (1976) and Silant'ev & Yakovlev (1980). In the degenerate regime, electrons dominate the conduction; we use the conductivities of Hernquist (1984) or Potekhin & Yakovlev (1996) and present results using both these values. In the semi-degenerate regime, both processes are important, so we sum the two conductivities.

Potekhin & Yakovlev (1996) give formulae to calculate electron conductivities in the liquid and solid regimes for arbitrary magnetic field strengths. The results of Hernquist (1984) are given for specific values of the field strength. We calculate the conductivities using the formalism and assumptions outlined in Hernquist (1984) and extend his calculations to stronger fields.

The conductivities of Hernquist (1984) and Potekhin & Yakovlev (1996) do not differ in the physical processes considered in their calculation but in the approximations employed. In the liquid state the conductivities of Hernquist (1984) tend to be approximately 15% larger for the ground Landau level and up to 40% larger for the excited levels than those of Potekhin & Yakovlev (1996). In the solid state, the conductivities of Potekhin & Yakovlev (1996) exceed those of Hernquist (1984) by a factor of several; therefore, these two models span much of the uncertainty in these quantities.

In the liquid state, the differences arise from two sources. First, both the fits of Hernquist (1984) and Potekhin & Yakovlev (1996) for the function $\phi(E)$ are inaccurate to $\sim 10\%$. Second, Hernquist (1984) assumes that electron-ion scattering is screened by the ion sphere; this process dominates in the liquid regime. Potekhin & Yakovlev (1996) include Debye and electron screening as well which dominate in the gaseous regime. Their results are appropriate for both the gaseous and liquid regimes. In the solid regime, Hernquist (1984) does not take the Debye-Waller factor into account. This factor tends to increase the conductivity over a wide range of temperatures and densities (Itoh et al. 1984, Potekhin & Yakovlev 1996).

Our iron envelope models are calculated by adopting a plane-parallel, Newtonian approximation. Hernquist (1985) found that using $Z = 26$ and $A = 56$ throughout is sufficient to accurately model the envelope. For simplicity we fix Z and A to these values, rather than use the equilibrium composition of Baym, Pethick & Sutherland (1971). In our approach, the core temperature is a function of F/g_s, B and ψ (the angle between the radial and field directions). Here, F is the transmitted heat flux, g_s is the surface gravity, and all of these values are taken to be in the frame of the neutron star surface – we have not applied the gravitational redshift to transform from the surface to the observer's frame.

For such strong fields, the models have a simple dependence on the angle ψ; *i.e.* $F/g_s \propto \cos^2 \psi$ (Greenstein & Hartke 1983, Page 1995, Shibanov et al. 1995, Shibanov & Yakovlev 1996, Heyl & Hernquist 1997) and furthermore the flux for a fixed core temperature is approximately proportional to $B^{0.4}$. With these two facts, we find that the average flux over the surface of a neutron star with a dipole field configuration is 0.4765 times its peak value at the magnetic poles. We neglect the effects of general relativity on the field configuration, which tend to make the field more radial (Ginzburg & Ozernoy 1964), increasing the effects discussed here.

Using these models, we have calculated a grid of theoretical envelopes with average effective temperatures ranging from $10^{5.4}$ K to $10^{6.6}$ K, corresponding to a factor of $\sim 10^5$ in transmitted flux. We

take $g_s = g_{s,14}10^{14}$ cm/s^2. In the upper panel, Figure 1 depicts the ratio of the core temperature to the zero-field case (Hernquist & Applegate 1984) as a function of the magnetic field and the mean effective temperature $\bar{T}_{\text{eff}}$ over the neutron star. In the zero-field case, to determine the core temperature for a given flux we combine equations (4.7) and (4.8) of Hernquist & Applegate (1984), switching from the first relation to the second when the surface effective temperature drops below 4.25×10^5 K. The results do not depend qualitatively on whether equation (4.7) or (4.8) of Hernquist & Applegate (1984) is used. The lower panel shows the ratio of the core temperature with a magnetic field to the zero-field case for the conductivities of Potekhin & Yakovlev (1996).

For a given core temperature, the magnetized envelopes transmit more heat than the unmagnetized envelopes. For example, an effective temperature of 3.5×10^6 K corresponds to a core temperature of 1.1×10^9 K for an unmagnetized envelope. With $B = 10^{16}$ G, the core temperature is 5.3×10^8 K for the Hernquist (1984) conductivities and 5.8×10^8 K for the Potekhin & Yakovlev (1996) ones. Because the Hernquist (1984) conductivities in the liquid phase are ~ 20 % larger than those of Potekhin & Yakovlev (1996), we find that the effective temperature is slightly higher during the early cooling ($t \lesssim 10^5$ yr) of the neutron star if one uses the values of Hernquist (1984).

For lower core temperatures, the insulating envelope is thinner and the magnetic field has a stronger effect (Van Riper 1988); the difference in core temperatures may be even more extreme, by up to a factor of four or ten (for Hernquist 1984 and Potekhin & Yakovlev 1996 conductivities, respectively) in the coolest envelopes considered here. For the cooler envelopes the relationship between the effective temperature and the core temperature is strongly sensitive to the conductivities in the solid phase; consequently, the Debye-Waller factor is most important during the later cooling of the neutron star ($t \gtrsim 10^5$ yr). Furthermore, during this late phase, the partial ionization of iron may affect the equation of state as well as the electron and photon conductivities. This area has not been thoroughly explored, especially at high B.

For $T_c > 10^8$ K, the dominant heat loss mechanism is through neutrino emission (*e.g.* Shapiro & Teukolsky 1983) which has a cooling time proportional to T_c^{-6} for the modified URCA process. Because of this steep power law, a factor of 1.9–2.1 difference in the core temperature (the first example) would lead one to infer a cooling time of an ultramagnetized neutron star 50–80 times greater than if one did not consider the effects of the magnetic field on heat transport. And since neutrino cooling models generically have a cooling time proportional to $T_c^{-\alpha}$ with $\alpha = 4 - 6$ (*e.g.* Shapiro & Teukolsky 1983), one would generally underestimate the cooling ages of ultramagnetized neutron stars by a large factor.

3. Thermal Evolution

For $t \gtrsim 10^3$ yr, the neutron star interior has relaxed thermally (Nomoto & Tsuruta 1981), we can use the flux-to-core-temperature relation for several values of B including $B = 0$ to derive the relationship between T_{eff} and the cooling time. The technique is straightforward during the epoch of neutrino cooling. However, since photon emission is enhanced, the epoch of photon cooling will begin slightly earlier, and the time dependence of the temperature will have a slightly different slope. For the neutrino cooling model, we use the modified URCA process (*e.g.* Shapiro & Teukolsky 1983)

$$L_\nu = (5.3 \times 10^{39}\text{erg/s})\frac{M}{M_\odot} \left(\frac{\rho_{\text{nuc}}}{\rho}\right)^{1/3} T_{c,9}^8 \tag{1}$$

where $T_x = T/10^x$ K.

To understand the evolution during the photon-cooling epoch, we take into account the surface thermal emission of photons,

$$L_\gamma = 4\pi R^2 \bar{T}_{\text{eff}}^4 \approx 9.5 \times 10^{32} \text{erg s}^{-1} \frac{\bar{T}_{\text{eff},6}^4}{g_{s,14}} \frac{M}{M_\odot} \tag{2}$$

and we take the total thermal energy of the neutron star to be (Shapiro & Teukolsky 1983)

$$U_n \simeq 6 \times 10^{47} \text{erg} \frac{M}{M_\odot} \left(\frac{\rho}{\rho_{\text{nuc}}}\right)^{-2/3} T_{c,9}^2. \tag{3}$$

Combining these equations yields

$$\frac{\mathrm{d}U_n}{\mathrm{d}t} = -(L_\nu + L_\gamma) \tag{4}$$

$$\frac{\mathrm{d}T_{c,9}}{\mathrm{d}t} = -\frac{1}{T_{c,9}} \left[\frac{1}{4 \times 10^7 \text{yr}} \frac{\bar{T}_{\text{eff},6}^4}{g_{s,14}} \left(\frac{\rho}{\rho_{\text{nuc}}}\right)^{2/3} + \frac{1}{8\text{yr}} \left(\frac{\rho_{\text{nuc}}}{\rho}\right)^{1/3} T_{c,9}^8 \right] \tag{5}$$

where ρ is the mean density of the neutron star, and $\rho_{\text{nuc}} = 2.8 \times 10^{14}$ g cm^{-3}.

Figure 2 shows the evolution of the core temperature and mean effective temperature at the surface. The evolution of the core temperature is unaffected by the magnetic field during the neutrino-cooling epoch. For fields approaching 10^{18} G, the magnetic field may begin to affect neutrino emission (*e.g.* Bander & Rubinstein 1993). However, after approximately $10^6 w$ yr, photon emission from the surface begins to dominate the evolution. Here,

$$w = \left(\frac{\rho}{\rho_{\text{nuc}}}\right)^{2/3} g_{s,14}^{-1}. \tag{6}$$

The cooling is accelerated by the magnetic field. In the presence of a 10^{16} G field, the core reaches a temperature of 10^7 K in only $3 - 6 \times 10^5 w$ yr compared to $6 \times 10^6 w$ yr for an unmagnetized neutron star. The bold curves trace the cooling of the core using the Hernquist (1984) conductivities, and the light curves follow the results for the Potekhin & Yakovlev (1996) conductivities.

The effect is more dramatic when one compares the effective surface temperatures of the models as a function of time. Again we present two sets of models. The lower panel compares the cooling evolution using the conductivities of Hernquist (1984) (bold curves) and using those of Potekhin & Yakovlev (1996) (light curves) in the degenerate regime.

During the neutrino-cooling epoch the ultramagnetized neutron stars ($B = 10^{16}$ G) have 45% higher effective temperatures and emit over four times more radiation. Because during neutrino cooling the effective temperature falls relatively slowly with time, one can make a large error in estimating the age of the neutron star from its luminosity. For example, an envelope with 10^{16} G field remains above a given effective temperature 40 times longer than an unmagnetized envelope. For 10^{15} G, the timescale is increased by up to a factor of ten.

During the photon-dominated cooling era, the enhanced flux in a strong magnetic field reverses this effect. Photon cooling begins to dominate after about $10^5 w$ yr for 10^{16} G compared to $10^6 w$ yr in the zero-field case. Once photon cooling begins to dominate, the stars with stronger magnetic fields cool more quickly. A star with a 10^{16} G field reaches a given effective temperature 3–5 times faster than an unmagnetized star.

4. Discussion

Magnetic fields, especially those associated with magnetars, have a strong effect on the observed thermal evolution of neutron stars. In agreement with Shibanov & Yakovlev (1996), we find that during the neutrino cooling epoch, neutron stars with strong magnetic fields are brighter than their unmagnetized coevals. During the photon cooling epoch, the situation is reversed. A strongly magnetized neutron star cools more quickly during this era and emits less radiation at a given age.

It is difficult to compare our results more quantitatively with those of Shibanov & Yakovlev (1996), because besides studying more weakly magnetized neutron stars, they make slightly different assumptions regarding the properties of the envelope, include general relativistic effects on the magnetic field geometry, and use the models of Van Riper (1988) which include Coulomb corrections to the equation of state. For the larger fields investigated here we do find a stronger effect than Shibanov & Yakovlev (1996); however, the effect is not as strong as a naive power-law extrapolation from $10^{13.5}$ G (the largest field studied by Shibanov & Yakovlev 1996) to the ultramagnetized regime would indicate. Usov (1997) also extrapolates results at lower field strengths and finds substantially larger photon luminosities during the neutrino-cooling epoch than we do. Again, a straightforward extrapolation from weaker fields overestimates the flux transmitted through a magnetized envelope.

We do not find the net insulating effect that Tsuruta & Qin (1995) find for weaker fields of 10^{12} G. Shibanov & Yakovlev (1996) find a similar, albeit much weaker effect, for fields $\sim 10^{10} - 10^{12}$ G. At these field strengths, the classical decrement in the thermal conductivity transverse to the field direction decreases the transmitted flux for a given core temperature. At the much stronger fields examined here, the increase in conductivity along the field lines (due to the quantization of the electron energies) dominates the decrease for perpendicular transport (in using the $\cos^2 \psi$ rule we have neglected all heat transport perpendicular to the field lines).

Thompson & Duncan (1995) argue that soft gamma repeaters (SGRs) are powered by magnetic reconnection events near the surfaces of ultramagnetized neutron stars. Furthermore, Ulmer (1994) finds that a strong magnetic field can explain the super-Eddington radiation transfer in SGRs. Rothschild, Kulkarni & Lingenfelter (1994) estimate the luminosity of SGR 0526-66 in the quiescent state to be approximately 7×10^{35} erg/sec. Since SGR 0526-66 is located in a supernova remnant, they can also estimate the age of the source to be approximately 5,000 years. For an isolated neutron star cooling by the modified URCA process, after 5,000 years, one would expect $L_\gamma = 6 \times 10^{33}$ erg/s for $B = 0$ and $L_\gamma = 3 \times 10^{34}$ erg/s for $B = 10^{16}$ G (we assume that the mass of the neutron star is $1.4 M_\odot$). Both these estimates fall short of the observed value. Even if SGR 0526-66 is powered by an ultramagnetized neutron star, its quiescent X-ray luminosity does not originate entirely from the thermal emission from the surface of the neutron star, unless either the age or luminosity estimates are in error by an order of magnitude, or possibly it has an accreted envelope.

5. Conclusions

We extend the previous studies of neutron star cooling into the ultramagnetized or magnetar regime ($B \sim 10^{15} - 10^{16}$ G) for iron envelopes. We find that such an intense magnetic field dramatically affects the thermal evolution of a neutron star. In the neutrino-cooling epoch, effective temperatures of ultramagnetized neutron stars are up to 40% larger than their unmagnetized coevals. If the nucleons in the neutron star core are superfluid, neutrino cooling is inhibited. This will also increase the surface temperature at a given

epoch.

Furthermore, if one assumes an unmagnetized evolutionary track for an ultramagnetized neutron star, one would overestimate its age by up to a factor of twenty five. During the photon-cooling epoch, the effect is reversed. Ultramagnetized neutron stars cool to a given effective temperature three times faster than their unmagnetized counterparts.

REFERENCES

Bander, M. & Rubinstein, H. 1993, *Phys Lett B,* **311**, 187.

Baym, G., Pethick, C. J. & Sutherland, P. G. 1971, *ApJ,* **170**, 299.

Duncan, R. C. & Thompson, C. 1992, *ApJL,* **392**, 9.

Ginzburg, V. L. & Ozernoy, L. M. 1964, *Zh. Exp. Teor. Fiz.,* **47**, 1030.

Greenstein, G. & Hartke, G. J. 1983, *ApJ,* **271**, 283.

Hernquist, L. & Applegate, J. H. 1984, *ApJ,* **287**, 244.

Hernquist, L. 1984, *ApJS,* **56**, 325.

Hernquist, L. 1985, *MNRAS,* **213**, 313.

Heyl, J. S. & Hernquist, L. *Almost Analtyic Models of Ultramagnetized Neutron Star Envelopes*, submitted to *MNRAS*

Itoh, N., Yasuharu, Kohyama, Matsumoto, N. & Seki, M. 1984, *ApJ,* **285**, 758.

Kippenhahn, R. & Weigert, A. 1990, *Stellar Structure and Evolution*, Springer, Berlin

Nomoto, K. & Tsuruta, S. 1981, *ApJL,* **250**, 19.

Page, D. 1995, *ApJ,* **442**, 273.

Pavlov, G. G. & Panov, A. N. 1976, *Sov. Phys. JETP,* **44**, 300.

Potekhin, A. Y. & Yakovlev, D. G. 1996, *A & A,* **314**, 341.

Rothschild, R. E., Kulkarni, S. R. & Lingenfelter, R. E. 1994, *Nature,* **368**, 432.

Shapiro, S. L. & Teukolsky, S. A. 1983, *Black Holes, White Dwarfs, and Neutron Stars*, Wiley-Interscience, New York

Shibanov, Y. A., Pavlov, G. G., Zavlin, V. E. & Tsuruta, S. 1995, in H. Böhringer, G. E. Morfill & J. E. Trümper (eds.), *Seventeenth Texas Symposium on Relativistic Astrophysics and Cosmology*, Vol. 759 of *Annals of the New York Academy of Sciences*, p. 291, The New York Academy of Sciences, New York

Shibanov, Y. A. & Yakovlev, D. G. 1996, *A&A,* **309**, 171.

Silant'ev, N. A. & Yakovlev, D. G. 1980, *Astrophys. Sp. Sci.,* **71**, 45.

Thompson, C. & Duncan, R. C. 1993, *ApJ,* **408**, 194.

Thompson, C. & Duncan, R. C. 1995, *MNRAS,* **275**, 255.

Tsuruta, S. & Qin, L. 1995, in H. Böhringer, G. E. Morfill & J. E. Trümper (eds.), *Seventeenth Texas Symposium on Relativistic Astrophysics and Cosmology*, Vol. 759 of *Annals of the New York Academy of Sciences*, p. 299, The New York Academy of Sciences, New York

Ulmer, A. 1994, *ApJL,* **437**, 111.

Usov, V. V. 1992, *Nature,* **357**, 472.

Usov, V. V. 1997, *A&A,* **317**, 87.

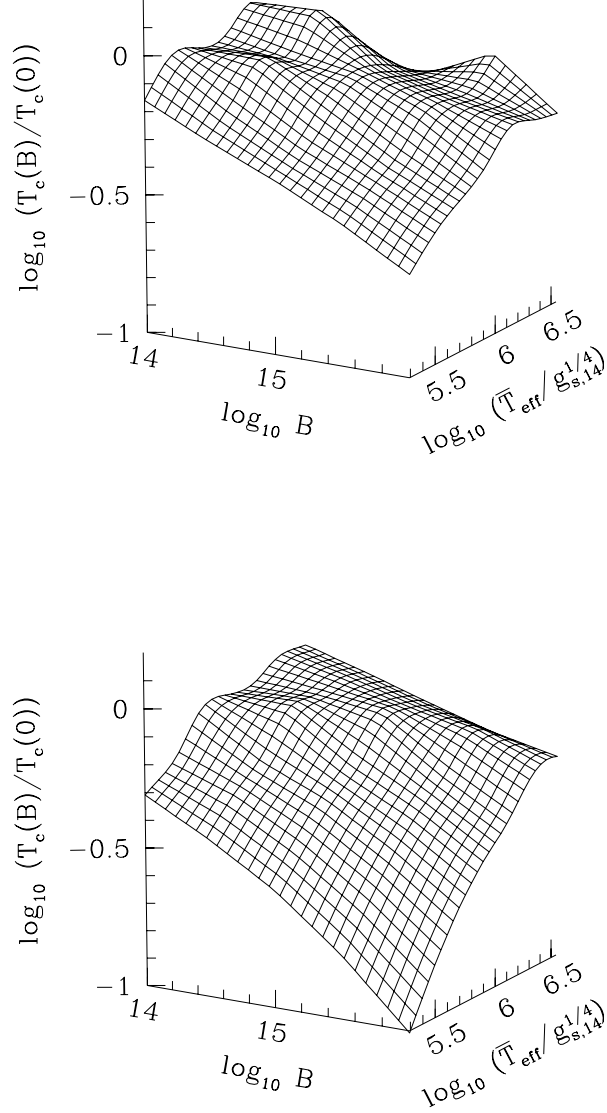Van Riper, K. A. 1988, *ApJ,* **329**, 339.

Fig. 1.— The upper panel depicts the ratio of the core temperature with $B \neq 0$ to the zero-field case (Hernquist & Applegate 1984) using the conductivities of Hernquist (1984). The lower panel shows the results for the Potekhin & Yakovlev (1996) conductivities.
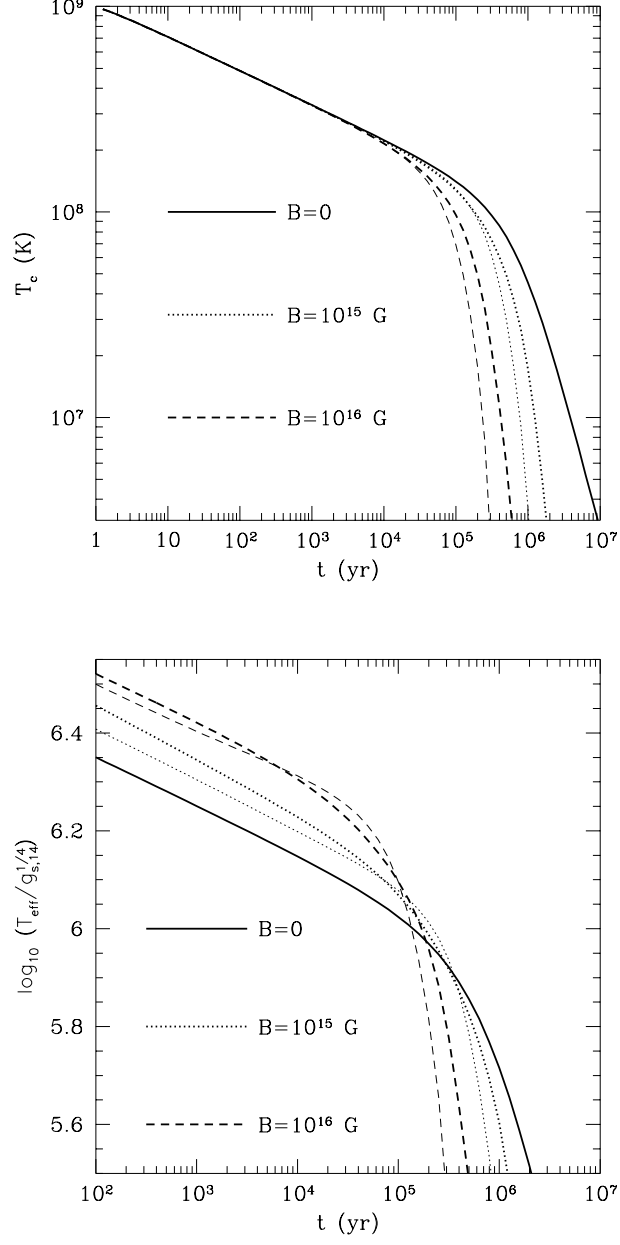
Fig. 2.— The upper panel depicts the evolution of the core temperature with time for neutrino-ominated cooling (independent of B) and photon-dominated cooling for several field strengths using the conductivities of Hernquist (1984) (bold curves) and Potekhin & Yakovlev (1996) (light curves). The lower panel shows the evolution of the mean effective temperature of the cooling neutron star for the neutrino and photon cooling epochs. The bold and light curves designate the same models as in the upper panel.